

Exercise 28

Find the derivative of the function.

$$f(z) = e^{z/(z-1)}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned} f'(z) &= \frac{df}{dz} = \frac{d}{dz} \left[e^{z/(z-1)} \right] \\ &= e^{z/(z-1)} \cdot \frac{d}{dz} \left(\frac{z}{z-1} \right) \\ &= e^{z/(z-1)} \cdot \frac{\left[\frac{d}{dz}(z) \right] (z-1) - \left[\frac{d}{dz}(z-1) \right] (z)}{(z-1)^2} \\ &= e^{z/(z-1)} \cdot \frac{(1)(z-1) - (1)(z)}{(z-1)^2} \\ &= e^{z/(z-1)} \cdot \frac{-1}{(z-1)^2} \\ &= -\frac{e^{z/(z-1)}}{(z-1)^2} \end{aligned}$$